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FSH 507 Spatio-temporal modeling

Homework 3 – Temporal models

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**Simulation experiment**

I used a dynamic linear model to simulate 100 replicates of observed data from three levels of autocorrelation crossed with three levels of measurement error, keeping process error fixed between scenarios. The estimation procedure matched the dynamic linear model used in the data generation. This simulation study tested the ability to estimate the autocorrelation, process error, and observation error in all scenarios of true autocorrelation and observation error.

I found that estimates of autocorrelation were unbiased regardless of the level of autocorrelation or true observation error. However, the precision of the estimates varied by level of observation error. With relatively low observation error, the precision was relatively high, but with relatively high observation error, the precision was lower. There were biases in estimates of process and observation error across all levels of autocorrelation. When there was no autocorrelation, estimates of process and observation error were unbiased only when the true level of observation error was high. When observation error was high, all parameter estimates were similarly imprecise. An interesting finding is that the estimates of autocorrelation were unbiased for all scenarios, regardless of whether the process and observation error estimates were biased. The unbiased estimation of autocorrelation is a benefit to the hierarchical structure of a dynamic linear model. Depending on the research question, it may be a problem that process and observation error are confounded and difficult to tease apart if these are direct parameters of interest.

Table 1. Mean, standard deviation, 2.5 percentile, and 97.5 percentile from 100 replicates of generated data given three levels of true autocorrelation and three levels of true observation error for the three parameters of the dynamic linear model. Standard errors are highest when observation error is highest. Estimates are unbiased for the autocorrelation parameter, but biases exist in the estimates of the process and observation error.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | autocorrelation (α) | | | process error (σp) | | | observation error (σy) | | |
| Mean | | | | | | | | | |
|  | Observation error | | | Observation error | | | Observation error | | |
| Autocorrelation | 0.2 | 0.4 | 0.8 | 0.2 | 0.4 | 0.8 | 0.2 | 0.4 | 0.8 |
| -0.5 | -0.501 | -0.481 | -0.432 | 0.145 | 0.158 | 0.239 | 0.048 | 0.155 | 0.546 |
| 0 | 0.006 | 0.017 | 0.021 | 0.119 | 0.169 | 0.399 | 0.074 | 0.141 | 0.374 |
| 0.5 | 0.512 | 0.504 | 0.477 | 0.154 | 0.168 | 0.231 | 0.040 | 0.145 | 0.553 |
| Standard deviation | | | | | | | | | |
|  | Observation error | | | Observation error | | | Observation error | | |
| Autocorrelation | 0.2 | 0.4 | 0.8 | 0.2 | 0.4 | 0.8 | 0.2 | 0.4 | 0.8 |
| -0.5 | 0.101 | 0.128 | 0.218 | 0.056 | 0.092 | 0.269 | 0.042 | 0.083 | 0.278 |
| 0 | 0.159 | 0.205 | 0.277 | 0.081 | 0.128 | 0.332 | 0.085 | 0.132 | 0.329 |
| 0.5 | 0.091 | 0.125 | 0.201 | 0.051 | 0.088 | 0.244 | 0.039 | 0.083 | 0.248 |
| 2.5 percentile | | | | | | | | | |
|  | Observation error | | | Observation error | | | Observation error | | |
| Autocorrelation | 0.2 | 0.4 | 0.8 | 0.2 | 0.4 | 0.8 | 0.2 | 0.4 | 0.8 |
| -0.5 | -0.704 | -0.718 | -0.740 | 0.043 | 0.015 | 0.000 | 0.000 | 0.000 | 0.001 |
| 0 | -0.312 | -0.337 | -0.482 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.001 |
| 0.5 | 0.339 | 0.258 | 0.055 | 0.046 | 0.031 | 0.000 | 0.000 | 0.000 | 0.001 |
| 97.5 percentile | | | | | | | | | |
|  | Observation error | | | Observation error | | | Observation error | | |
| Autocorrelation | 0.2 | 0.4 | 0.8 | 0.2 | 0.4 | 0.8 | 0.2 | 0.4 | 0.8 |
| -0.5 | -0.332 | -0.233 | 0.097 | 0.240 | 0.362 | 0.855 | 0.136 | 0.300 | 0.976 |
| 0 | 0.287 | 0.358 | 0.487 | 0.227 | 0.374 | 0.932 | 0.234 | 0.389 | 0.932 |
| 0.5 | 0.668 | 0.707 | 0.823 | 0.241 | 0.369 | 0.906 | 0.132 | 0.311 | 0.929 |

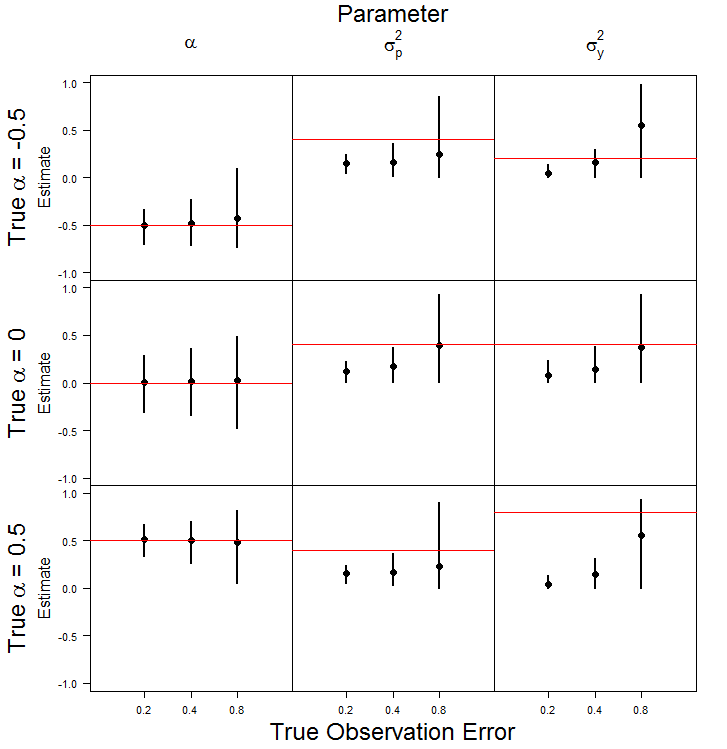


Figure 1. Mean estimates of the three parameters for the dynamic linear model from 100 replicates of simulated data for each of three levels of observation error and three levels of true autocorrelation. Line segments represent the 2.5 and 97.5 percentile from the 100 replicates of simulated data.

**Gompertz model**

When generating data using a Gompertz model, I ensured the population had hit equilibrium. This could be done analytically, but I displayed the visual confirmation in Figure 1. By starting the time series at a point away from the equilibrium point, the density-dependent nature of the Gompertz model brings the value to an equilibrium after less than 100 observations. The data I used to run the model was the final 100 observations in a time series of 1000 observations. The Gompertz model may be more useful than the dynamic linear model when density-dependent processes are acting on the population.

It is useful to see that the autocorrelation parameters are unbiased, as we found with the dynamic linear model in the previous example. However, the standard deviation parameters are still confounded, and thus still biased. This exercise demonstrates that it is difficult to tease apart process and observation error. This type of model is very successful in identifying autocorrelation. However, if the level of process error separate from observation error is desired, these models would not be very useful.

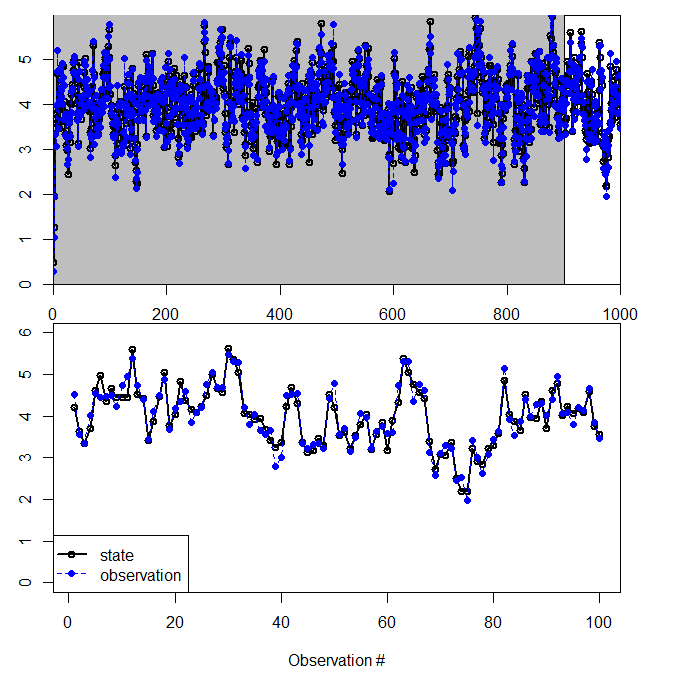


Figure 2. Plot of time series of the states (black) and observations (blue) generated by the Gompertz model. The top panel is 1000 observations used to visually confirm that the Gompertz model reached equilibrium. The bottom panel is the last 100 observations chosen as generated data once the model had reached equilibrium.

Table 2. Gompertz model parameter estimates and their associated standard deviations.

|  |  |  |
| --- | --- | --- |
| Parameter | Mean | Standard Deviation |
| α | 1.340 | 0.307 |
| β | 0.663 | 0.075 |
| σp | 0.544 | 0.039 |
| σy | 0.006 | 0.132 |